VI Всероссийская школа-семинар по электромагнитным зондированиям Земли

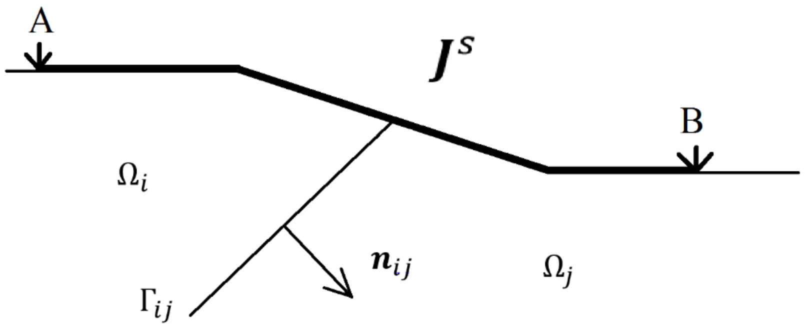
имени М.Н. Бердичевского и Л.Л. Ваньяна

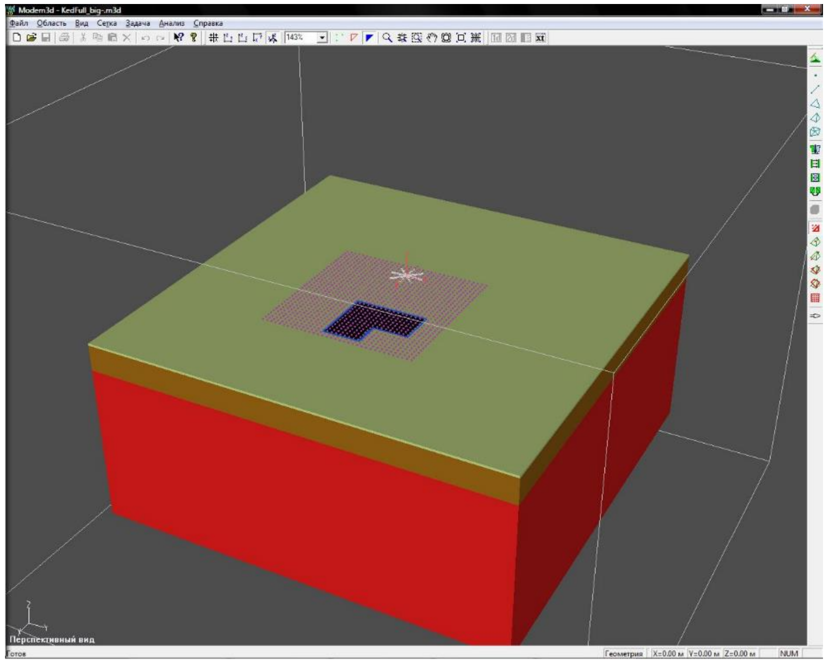
ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧ ГЕОЭЛЕКТРИКИ В НЕОДНОРОДНЫХ 3D ОБЛАСТЯХ С РЕЛЬЕФОМ

Иванов М.И., Кремер И.А.

ИНГГ СО РАН,   г. Новосибирск

ЗАО «Центр РИТМ»,   г. Новосибирск





$$\Omega \subset \mathbb{R}^3 \qquad \Omega = \bigcup_i \Omega_i \qquad \begin{matrix} \mu|_{\Omega_i} = \mu_i & \mu_i > 0 \\ \sigma|_{\Omega_i} = \sigma_i & \sigma_i > 0 \end{matrix}$$

Уравнения в подобластях

$$rot\, \boldsymbol{H}|_{\Omega_i} = (\sigma_i \boldsymbol{E} + \boldsymbol{J}^s)|_{\Omega_i}, \quad div\, (\sigma_i \boldsymbol{E} + \boldsymbol{J}^s)|_{\Omega_i} = 0,$$

$$rot\, \boldsymbol{E} + \mu_i \frac{\partial \boldsymbol{H}}{\partial t}\bigg|_{\Omega_i} = \boldsymbol{0}, \qquad div\, \mu_i \boldsymbol{H}|_{\Omega_i} = 0.$$

Условия сопряжения

$$[\boldsymbol{n}_{ij} \times \boldsymbol{E}]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\sigma \boldsymbol{E} \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = 0,$$

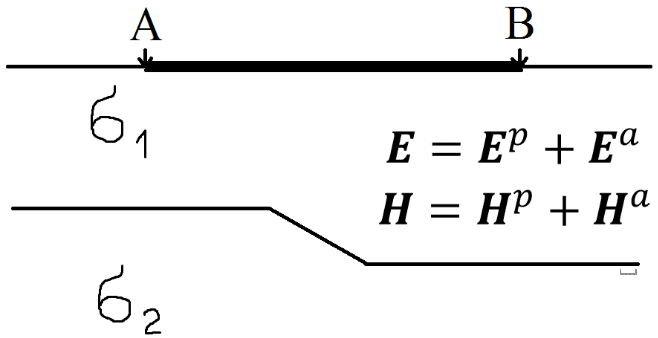
$$[\boldsymbol{n}_{ij} \times \boldsymbol{H}]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\mu \boldsymbol{H} \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = 0,$$

Краевые условия

$$\boldsymbol{n} \times \boldsymbol{E}|_{\Gamma} = \boldsymbol{0}, \quad \boldsymbol{H} \cdot \boldsymbol{n}|_{\Gamma} = 0$$

Начальные данные

$$\boldsymbol{E}|_{t=0} = \boldsymbol{E}_0, \quad \boldsymbol{H}|_{t=0} = \boldsymbol{H}_0.$$



$$\boldsymbol{E} = \boldsymbol{E}^p + \boldsymbol{E}^a$$
$$\boldsymbol{H} = \boldsymbol{H}^p + \boldsymbol{H}^a$$

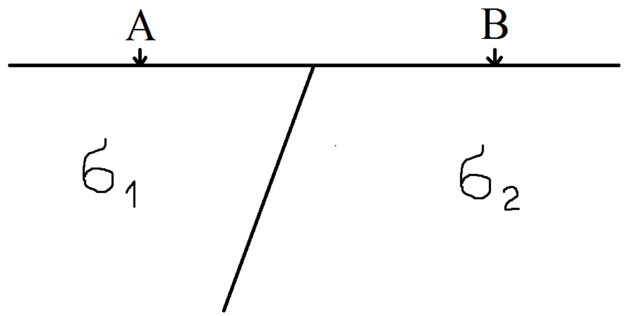
$$\boldsymbol{E} = \boldsymbol{E}^p + \boldsymbol{E}^a, \quad \boldsymbol{H} = \boldsymbol{H}^p + \boldsymbol{H}^a.$$

Первичные поля

$$rot\ \boldsymbol{H}^p = \sigma_1 \boldsymbol{E}^p + \boldsymbol{J}^s, \quad div\ (\sigma_1 \boldsymbol{E}^p + \boldsymbol{J}^s) = 0,$$
$$rot\ \boldsymbol{E}^p + \mu_1 \frac{\partial \boldsymbol{H}^p}{\partial t} = \boldsymbol{0}, \quad div\ \mu_1 \boldsymbol{H}^p = 0.$$
$$\boldsymbol{E}^p|_\infty = \boldsymbol{0}, \quad \boldsymbol{H}^p|_\infty = \boldsymbol{0}.$$
$$\boldsymbol{E}^p|_{t=0} = \boldsymbol{E}_0^p, \quad \boldsymbol{H}^p|_{t=0} = \boldsymbol{H}_0^p.$$

Аномальные поля

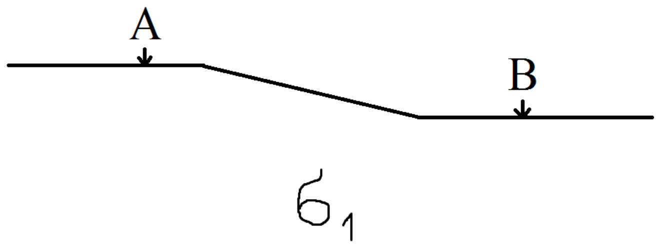
$$rot\ \boldsymbol{H}^a|_{\Omega_i} = (\sigma_i \boldsymbol{E}^a + (\sigma_i - \sigma_1)\boldsymbol{E}^p)|_{\Omega_i}, \quad div\ (\sigma_i \boldsymbol{E}^a + (\sigma_i - \sigma_1)\boldsymbol{E}^p)|_{\Omega_i} = 0,$$
$$rot\ \boldsymbol{E}^a + \mu_i \frac{\partial \boldsymbol{H}^a}{\partial t} + (\mu_i - \mu_1)\frac{\partial \boldsymbol{H}^p}{\partial t}\bigg|_{\Omega_i} = \boldsymbol{0}, \quad div\ (\mu_i \boldsymbol{H}^a + (\mu_i - \mu_1)\boldsymbol{H}^p)|_{\Omega_i} = 0.$$
$$[\boldsymbol{n}_{ij} \times \boldsymbol{E}^a]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\sigma \boldsymbol{E}^a \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = -[\sigma \boldsymbol{E}^p \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}},$$
$$[\boldsymbol{n}_{ij} \times \boldsymbol{H}^a]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\mu \boldsymbol{H}^a \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = -[\mu \boldsymbol{H}^p \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}}.$$
$$\boldsymbol{n} \times \boldsymbol{E}^a|_\Gamma = -\boldsymbol{n} \times \boldsymbol{E}^p|_\Gamma, \quad \boldsymbol{H}^a \cdot \boldsymbol{n}|_\Gamma = -\boldsymbol{H}^p \cdot \boldsymbol{n}|_\Gamma.$$
$$\boldsymbol{E}^a|_{t=0} = \boldsymbol{E}_0^a, \quad \boldsymbol{H}^a|_{t=0} = \boldsymbol{H}_0^a.$$



$$\mu \boldsymbol{H} = rot\, \boldsymbol{A}, \qquad div\, \sigma \boldsymbol{A} = g.$$

$$-div\, \sigma_i \nabla U|_{\Omega_i} = -div\, \boldsymbol{J}^s|_{\Omega_i}.$$

$$[U]_{\Gamma_{ij}} = 0, \qquad [\sigma \nabla U \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = 0.$$

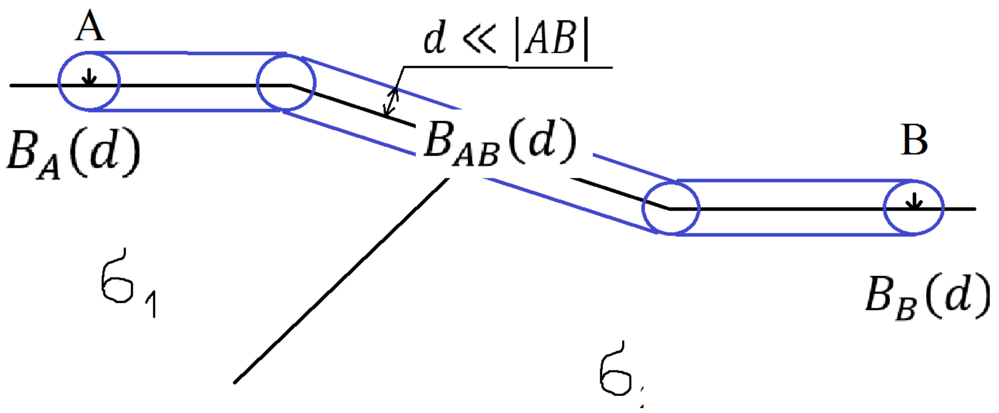


$$rot \frac{1}{\mu} rot\, \boldsymbol{A}\Big|_{\Omega_i} = (-\sigma_i \nabla U + \boldsymbol{J}^s)|_{\Omega_i},$$

$$div\, \sigma_i \boldsymbol{A}|_{\Omega_i} = g|_{\Omega_i}.$$

$$[\boldsymbol{n}_{ij} \times \boldsymbol{A}]_{\Gamma_{ij}} = \boldsymbol{0}, \quad \left[\boldsymbol{n}_{ij} \times \frac{1}{\mu} rot\, \boldsymbol{A}\right]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\sigma \boldsymbol{A} \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = 0,$$

$$\boldsymbol{n} \times \boldsymbol{A}|_{\Gamma} = \boldsymbol{0}, \quad U|_{\Gamma} = 0.$$



$$\boldsymbol{A} = \boldsymbol{A}^p + \boldsymbol{A}^a, \quad U = U^p + U^a.$$

$$supp\ \boldsymbol{A}^p \subset B_{AB}(d) \qquad supp\ U^p \subset B_A(d) \cup B_B(d)$$

$$-div\ \sigma_A \nabla U^p|_{B_A(d)} = -div\ \boldsymbol{J}^s|_{B_A(d)} \qquad -div\ \sigma_B \nabla U^p|_{B_B(d)} = -div\ \boldsymbol{J}^s|_{B_B(d)}.$$

$$U^p|_{\partial B_A(d)} = 0 \qquad U^p|_{\partial B_B(d)} = 0.$$

$$rot \frac{1}{\mu} rot\ \boldsymbol{A}^p \Big|_{B_i} = (-\sigma_i \nabla U^p + \boldsymbol{J}^s)|_{B_i},$$

$$div\ \sigma_i \boldsymbol{A}^p|_{B_i} = g|_{B_i}.$$

$$[\boldsymbol{n}_{ij} \times \boldsymbol{A}^p]_{\Gamma_{ij}} = \boldsymbol{0}, \quad \left[\boldsymbol{n}_{ij} \times \frac{1}{\mu} rot\ \boldsymbol{A}^p\right]_{\Gamma_{ij}} = \boldsymbol{0}, \quad [\sigma \boldsymbol{A}^p \cdot \boldsymbol{n}_{ij}]_{\Gamma_{ij}} = 0.$$

$$\boldsymbol{n} \times \boldsymbol{A}^p|_{\partial B_{AB}(d)} = \boldsymbol{0}.$$

$$-div\ \sigma_i \nabla U^a|_{\Omega_i} = 0.$$

$$[U^a]_{\Gamma_{ij}} = 0, \quad \left[\sigma \nabla U^a \cdot \boldsymbol{n}_{ij}\right]_{\Gamma_{ij}} = 0, \ \ U^a|_{\Gamma} = 0$$

$$rot \frac{1}{\mu} rot\ \boldsymbol{A}^a \Big|_{\Omega_i} = (-\sigma_i \nabla U^a)|_{\Omega_i},$$

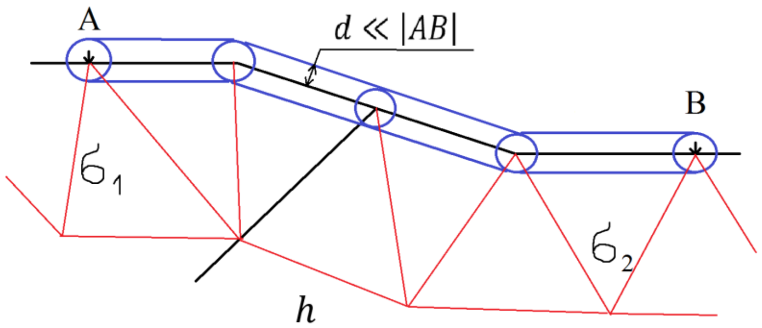
$$div\ \sigma_i \boldsymbol{A}^a|_{\Omega_i} = 0.$$

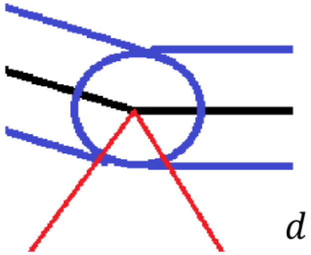
$$\left[\boldsymbol{n}_{ij} \times \boldsymbol{A}^a\right]_{\Gamma_{ij}} = \boldsymbol{0}, \quad \left[\boldsymbol{n}_{ij} \times \frac{1}{\mu} rot\ \boldsymbol{A}^a\right]_{\Gamma_{ij}} = \boldsymbol{0}, \quad \left[\sigma \boldsymbol{A}^a \cdot \boldsymbol{n}_{ij}\right]_{\Gamma_{ij}} = 0, \quad \boldsymbol{n} \times \boldsymbol{A}^a|_{\Gamma} = \boldsymbol{0}$$

Дополнительные условия на границе $\partial B_{AB}(d)$

$$[\sigma \nabla U^a \cdot \boldsymbol{n}_d]_{\partial B_A(d)} = \sigma \nabla U^p \cdot \boldsymbol{n}_d|_{\partial B_A(d)}, \qquad [\sigma \nabla U^a \cdot \boldsymbol{n}_d]_{\partial B_B(d)} = \sigma \nabla U^p \cdot \boldsymbol{n}_d|_{\partial B_B(d)}.$$

$$\left[\boldsymbol{n}_d \times \frac{1}{\mu} rot\ \boldsymbol{A}^a\right]_{\partial B_{AB}(d)} = \boldsymbol{n}_d \times \frac{1}{\mu} rot\ \boldsymbol{A}^p \Big|_{\partial B_{AB}(d)}, \qquad \left[\sigma \boldsymbol{A}^a \cdot \boldsymbol{n}_{ij}\right]_{\partial B_{AB}(d)} = \sigma \boldsymbol{A}^p \cdot \boldsymbol{n}_d|_{\partial B_{AB}(d)}.$$





$$T_h = \{K_i,\ \ i = 1,2,\dots,N_h\},\qquad \Omega = \cup_{i=1}^{N_h} K_i,\qquad d \ll h \le |AB|.$$

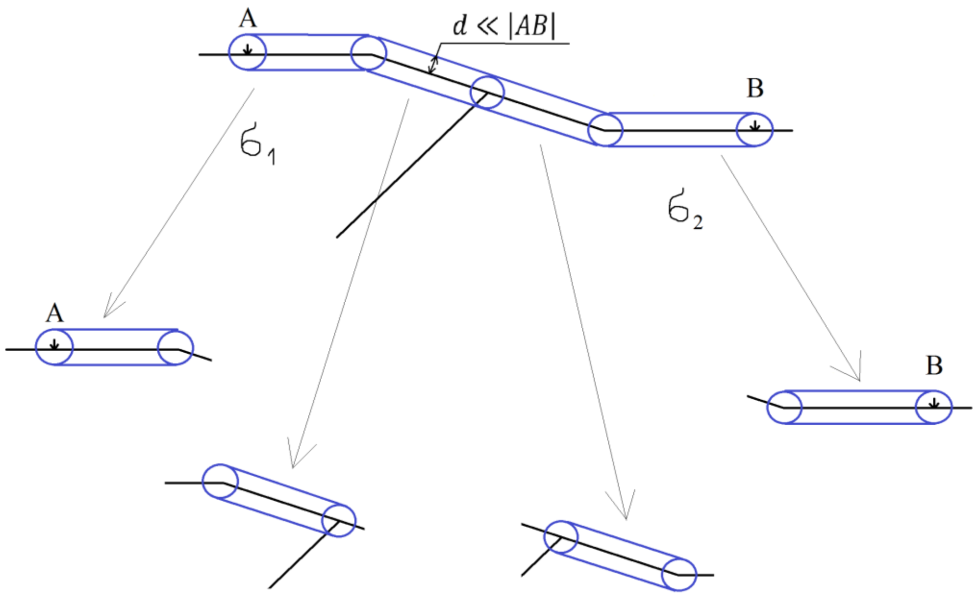
$$\boldsymbol{V}_h = span\,\{\boldsymbol{N}_j,\ \ j = 1,2,\dots,N_{\boldsymbol{V}}\},\qquad Q_h = span\,\{\varphi_k,\ \ k = 1,2,\dots,N_Q\}.$$

$$\int_{\partial B_{AB}(d)} \boldsymbol{n}_d \times \frac{1}{\mu} rot\ \boldsymbol{A}^p \cdot \boldsymbol{N}_j dS = \sum_{K\in T_h} \int_{K\cap\partial B_{AB}(d)} \boldsymbol{n}_d \times \frac{1}{\mu} rot\ \boldsymbol{A}^p \cdot \boldsymbol{N}_j dS\,,\ \ j = 1,2,\dots,N_{\boldsymbol{V}},$$

$$\int_{\partial B_{AB}(d)} \sigma\varphi_k\ \boldsymbol{A}^p \cdot \boldsymbol{n}_d dS = \sum_{K\in T_h} \int_{K\cap\partial B_{AB}(d)} \varphi_k\ \sigma\boldsymbol{A}^p \cdot \boldsymbol{n}_d dS\,,\qquad k = 1,2,\dots,N_Q,$$

$$\int_{\partial B_{A}(d)} \sigma\varphi_k\ \nabla U^p \cdot \boldsymbol{n}_d dS = \sum_{K\in T_h} \int_{K\cap\partial B_{A}(d)} \varphi_k\ \sigma\nabla U^p \cdot \boldsymbol{n}_d dS\,,\qquad k = 1,2,\dots,N_Q,$$

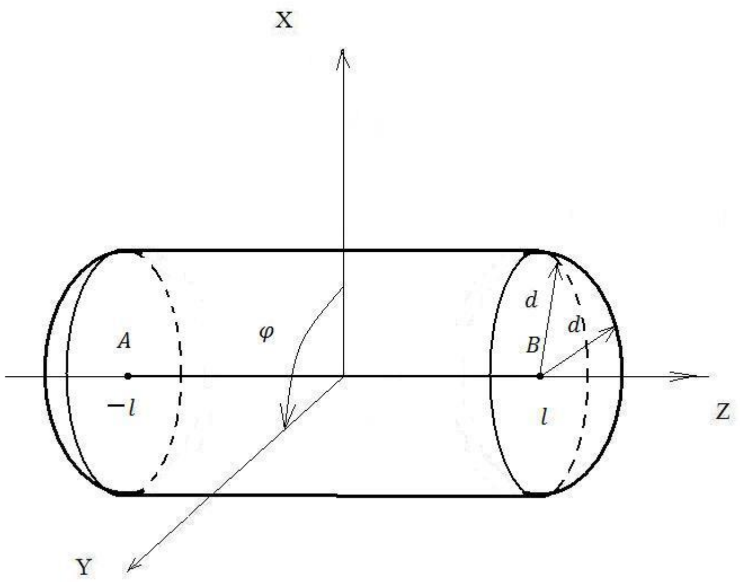
$$\int_{\partial B_{B}(d)} \sigma\varphi_k\ \nabla U^p \cdot \boldsymbol{n}_d dS = \sum_{K\in T_h} \int_{K\cap\partial B_{B}(d)} \varphi_k\ \sigma\nabla U^p \cdot \boldsymbol{n}_d dS\,,\qquad k = 1,2,\dots,N_Q.$$



$$U^p = \sum_i U_i^p, \qquad \boldsymbol{A}^p = \sum_i \boldsymbol{A}_i^p.$$

$$supp\ U_i^p \subset B_{A_i}(d) \cup B_{B_i}(d) \qquad supp\ \boldsymbol{A}_i^p \subset B_{A_iB_i}(d)$$

$$U^p|_{B_{A_i}(d) \cup B_{B_i}(d)} = U_i^p\Big|_{B_{A_i}(d) \cup B_{B_i}(d)} \qquad \boldsymbol{A}^p|_{B_{A_iB_i}(d)} = \boldsymbol{A}_i^p\Big|_{B_{A_iB_i}(d)}$$



$$\sigma_A = \sigma_B = \sigma_{AB} = \sigma, \qquad \mu_A = \mu_B = \mu_{AB} = \mu.$$

$$\boldsymbol{J}^s = (0, 0, J_z), \qquad J_z = I\,\big(\eta(z+l) - \eta(z-l)\big)\,\delta(x)\,\delta(y).$$

$$-div\,\sigma_A \nabla U^p|_{B_A(d)} = I\,\delta(z-l)\,\delta(x)\,\delta(y) \qquad -div\,\sigma_B \nabla U^p|_{B_B(d)} = -I\,\delta(z+l)\,\delta(x)\,\delta(y)$$

$$U^p|_{\partial B_A(d)} = 0 \qquad U^p|_{\partial B_B(d)} = 0$$

$$U^p(r)|_{B_A(d)} = \frac{I}{4\pi\sigma}\left(\frac{1}{r} - \frac{1}{d}\right) \qquad U^p(r)|_{B_B(d)} = -\frac{I}{4\pi\sigma}\left(\frac{1}{r} - \frac{1}{d}\right).$$

$$\sigma\nabla U^p \cdot \boldsymbol{n}_d|_{\partial B_A(d)} = -\frac{I}{4\pi d^2} \qquad \sigma\nabla U^p \cdot \boldsymbol{n}_d|_{\partial B_B(d)} = \frac{I}{4\pi d^2}.$$

$$rot\frac{1}{\mu}rot\,\boldsymbol{A}^p\Big|_{B_{AB}(d)} = (-\sigma\nabla U^p + \boldsymbol{J}^s)|_{B_{AB}(d)}$$

$$\boldsymbol{n} \times \boldsymbol{A}^a|_{\partial B_{AB}(d)} = \boldsymbol{0}$$

$$\boldsymbol{A}^p(\boldsymbol{r}) = \boldsymbol{A}_A^p(\boldsymbol{r}) + \boldsymbol{A}_B^p(\boldsymbol{r}) + \boldsymbol{A}_{AB}^p(\boldsymbol{r})$$

$$\boldsymbol{A}_A^p(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_{B_A(d)}\frac{-\sigma\nabla U^p(\boldsymbol{\xi})d\boldsymbol{\xi}}{|\boldsymbol{r}-\boldsymbol{\xi}|} \qquad \boldsymbol{A}_B^p(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_{B_B(d)}\frac{-\sigma\nabla U^p(\boldsymbol{\xi})d\boldsymbol{\xi}}{|\boldsymbol{r}-\boldsymbol{\xi}|} \qquad \boldsymbol{A}_{AB}^p(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_A^B\frac{\boldsymbol{J}^s(\boldsymbol{\xi})d\boldsymbol{\xi}}{|\boldsymbol{r}-\boldsymbol{\xi}|}.$$

$$(\rho,\theta,\varphi) \qquad \boldsymbol{A}_A^p(\boldsymbol{r}) = \left(A_\rho^p(\boldsymbol{r}),0,0\right) \qquad A_\rho^p(\boldsymbol{r}) = -\frac{\mu I}{4\pi}\left(1+\ln\frac{d}{r}\right) \qquad \boldsymbol{n}_d \times \boldsymbol{A}_A^p(\boldsymbol{r})\Big|_{\partial B_A(d)} \equiv \boldsymbol{0}.$$

$$\boldsymbol{A}_A^p(\boldsymbol{r})\cdot\boldsymbol{n}_d\Big|_{\partial B_A(d)} = -\frac{\mu I}{4\pi} \qquad \boldsymbol{n}_d \times rot\,\boldsymbol{A}_A^p(\boldsymbol{r})\Big|_{\partial B_A(d)} = \boldsymbol{0}$$

$$\boldsymbol{A}_B^p(\boldsymbol{r})\cdot\boldsymbol{n}_d\Big|_{\partial B_B(d)} = \frac{\mu I}{4\pi} \qquad \boldsymbol{n}_d \times rot\,\boldsymbol{A}_A^p(\boldsymbol{r})\Big|_{\partial B_B(d)} = \boldsymbol{0}$$

(ρ, φ, z) $\boldsymbol{A}_{AB}^{p}(\boldsymbol{r}) = \left(0, 0, A_z^p(\boldsymbol{r})\right)$ $A_z^p(\boldsymbol{r}) = \frac{\mu I}{4\pi} \ln\left(\frac{z + l + \sqrt{r^2 + (z+l)^2}}{z - l + \sqrt{r^2 + (z-l)^2}}\right)$ $\boldsymbol{n}_d \times \boldsymbol{A}_{AB}^{p}(\boldsymbol{r})\big|_{\partial B_{AB}(d)} \neq \boldsymbol{0}$

$\boldsymbol{f}_{AB}(\boldsymbol{r}) = \left(0, 0, f_z(\boldsymbol{r})\right)$

$$f_z^A = \begin{cases} \frac{\mu I}{4\pi} \ln\left(\frac{z + l + d}{z - l + \sqrt{d^2 - 4zl}}\right) & z \in [-l-d, -l[, \\ \frac{\mu I}{4\pi} \ln\left(\frac{z + l + \sqrt{d^2 + (z+l)^2}}{z - l + \sqrt{d^2 + (z-l)^2}}\right) & z \in [-l, l], \\ \frac{\mu I}{4\pi} \ln\left(\frac{z + l + \sqrt{d^2 + 4zl}}{z - l + d}\right) & z \in]l, l+d]. \end{cases}$$

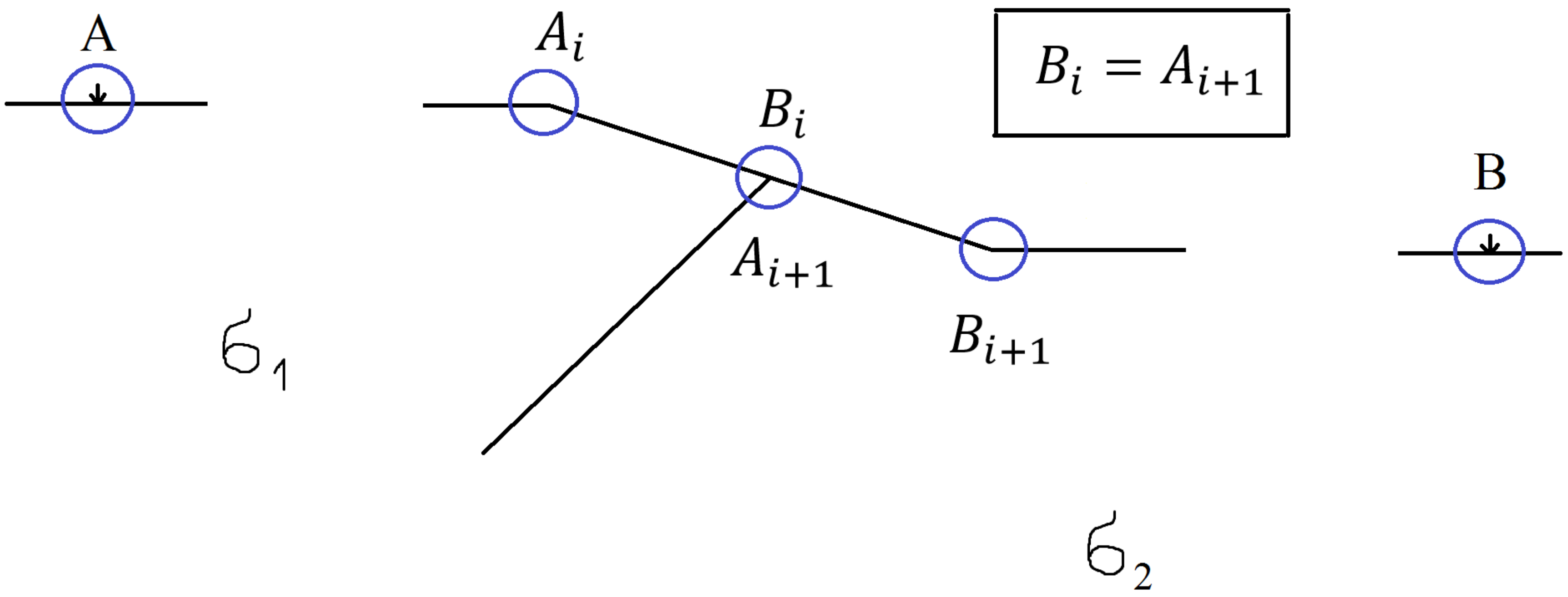
$rot\, \boldsymbol{f}_{AB}(\boldsymbol{r}) \equiv \boldsymbol{0}$ $\boldsymbol{n}_d \times \left(\boldsymbol{A}_{AB}^{p}(\boldsymbol{r}) - \boldsymbol{f}_{AB}(\boldsymbol{r})\right)\Big|_{\partial B_{AB}(d)} = \boldsymbol{0}.$

$$\left(\boldsymbol{A}_{AB}^{p}(\boldsymbol{r}) - \boldsymbol{f}_{AB}(\boldsymbol{r})\right) \cdot \boldsymbol{n}_d\Big|_{\partial B_{AB}(d)} = A_\rho^p(\boldsymbol{r})\big|_{\partial B_{AB}(d)} = -\frac{\mu I}{4\pi}$$

$$\boldsymbol{n}_d \times rot\, \left(\boldsymbol{A}_{AB}^{p}(\boldsymbol{r}) - \boldsymbol{f}_{AB}(\boldsymbol{r})\right)\Big|_{\partial B_{AB}(d)} = \boldsymbol{0}$$

$$\boldsymbol{A}^{p}(\boldsymbol{r}) = \boldsymbol{A}_{A}^{p}(\boldsymbol{r}) + \boldsymbol{A}_{B}^{p}(\boldsymbol{r}) + \boldsymbol{A}_{AB}^{p}(\boldsymbol{r}) - \boldsymbol{f}_{AB}(\boldsymbol{r})$$

$$div\, \sigma \boldsymbol{A}^p = g \neq 0$$

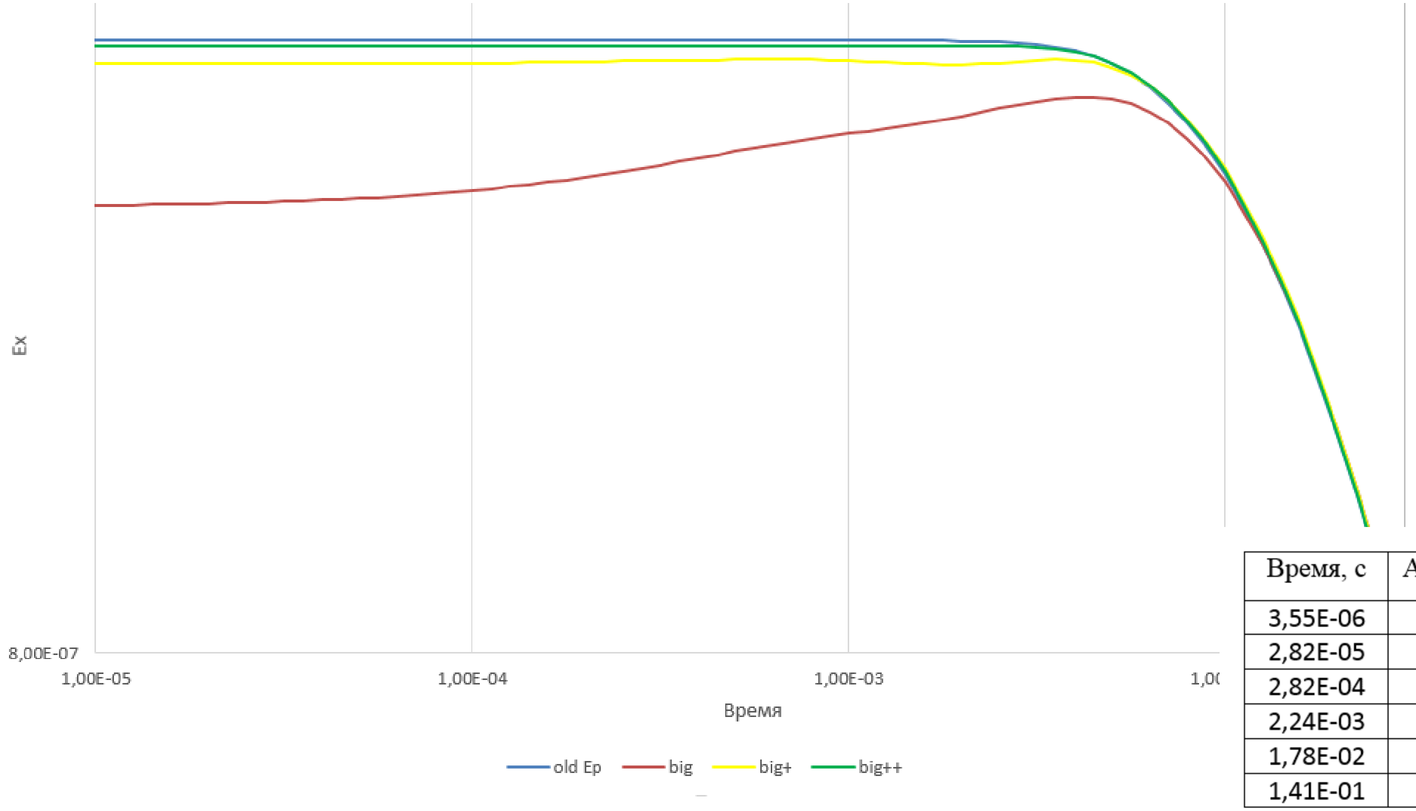


$$\boldsymbol{A}^{p}_{A_{i+1}}(\boldsymbol{r}) = -\boldsymbol{A}^{p}_{B_i}(\boldsymbol{r})$$

$$\Omega = 25\ 600\text{м.} \times 25\ 600\text{м.} \times 19\ 000\text{м.} \qquad \rho = 1\ \text{ом. м.}$$

$$|AB| = 500\ \text{м} \qquad I_{AB} = 1A$$

Имя	h, м	$N_{\text{ребер}}$	$N_{\text{узлов}}$	Ошибка, %
Big	125	705 908	116 045	17
Big+	62,5	1 009 552	160 569	3
Big++	31,25	1 221 836	191 301	1



Время, с	Аналитика	Big		Big+		Big++	
3,55E-06	1,61E-06	1,33E-06	-17%	1,57E-06	-3%	1,60E-06	-1%
2,82E-05	1,61E-06	1,34E-06	-17%	1,57E-06	-3%	1,60E-06	-1%
2,82E-04	1,61E-06	1,39E-06	-14%	1,57E-06	-2%	1,60E-06	-1%
2,24E-03	1,61E-06	1,48E-06	-8%	1,57E-06	-3%	1,60E-06	-1%
1,78E-02	1,09E-06	1,10E-06	1%	1,10E-06	1%	1,09E-06	0%
1,41E-01	1,64E-07	1,63E-07	-1%	1,63E-07	-1%	1,63E-07	-1%

$$\Omega = 25\,600\text{м.} \times 25\,600\text{м.} \times 19\,000\text{м.} \qquad \rho_1 = 1 \text{ ом. м.} \qquad \rho_2 = 10 \text{ ом. м.}$$

$$|AB| = 500 \text{ м} \qquad I_{AB} = 1A$$